## THERMAL-WAVE PROPAGATION

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UDC 536.45:533.6.011

It is shown in [1] that the internal shock wave has a significant effect on the conditions of thermal-wave propagation. In this paper, the effect of the mass  $M_0$  released by the thermal-wave propagation is estimated by a method similar to that in [1].

Suppose that, at the initial instant, energy  $E_0$  and mass  $M_0$  are released instantaneously in an infinitely small volume  $V_0$  and that the density of the energy and substance released is many times greater than the density of the energy and substance of the surrounding medium. A spherical shock wave propagates from the site of the explosion. In consequence of the presence of a pressure drop at the boundary between the mass  $M_0$  and the surrounding air, the movement of gas commences. The shock wave is propagating inside the thermal wave in an ideal gas defined by effective values of the adiabatic index  $\gamma_e$  and the gas constant  $A_e$ .

The system of equations describing the wave propagation processes has the form

$$\frac{\partial \rho}{\partial t} = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v); \qquad (1)$$

$$\frac{\partial}{\partial t}(\rho v) = -\frac{\partial}{\partial r}(\rho v^2 + \rho a^2) - \frac{2\rho v^2}{r}; \qquad (2)$$

$$\frac{\partial}{\partial t}\left(\frac{\rho a^2}{\gamma_e - 1} + \frac{\rho v^2}{2}\right) = -\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\rho v\left(\frac{a^2}{\gamma_e - 1} + \frac{v^2}{2} + a^2\right) - r^2S\right],\tag{3}$$

where  $S = -(16/3)\lambda R \sigma T^3 \partial T/\partial r$  is the radiation flow;  $a = \sqrt{A_e T}$  is the isothermal velocity of sound.

Equation (3), taking into account Eqs. (1) and (2), can be reduced to the usual form for writing the thermal-wave equation

$$\frac{A_{\rm e}\rho}{\gamma_{\rm e}-1} \frac{dT}{dt} - A_{\rm e}T \frac{d\rho}{dt} = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 S). \tag{4}$$

The boundary and initial conditions to systems (1)-(4) are written in the form

$$v(0, t) = 0, v(r \to \infty, t) = 0,$$
  

$$\rho(r \to \infty, t) = \rho_0, T(r \to \infty, t) = 0,$$
  

$$\lim_{V_0 \to 0} \int_{V_0} \rho(r, 0) d\mathbf{r} = M_0,$$
  

$$\lim_{V_0 \to 0} \int_{V_0} \frac{\rho(r, 0) A_e T(r, 0)}{\gamma_e - 1} d\mathbf{r} = E_0.$$
(5)

As the initial stage of growth of the high-temperature region, taking place at the expense of nonequilibrium radiation [2] during the time  $T_n$  ( $T_n \sim 10^{-7}$  sec), is much less than the characteristic time of the radiative thermal conductivity stage  $T_r \sim 10^{-5}-10^{-4}$  sec, we can substitute the last of conditions (5) by the relations

$$\lim_{V_0 \to 0} \int_{V_0}^{\rho v^2} d\mathbf{r} = \alpha E_0, \quad 0 < \alpha < 1,$$

$$4\pi \int_{0}^{r_{T_0}} \frac{\rho A_{e} T}{\gamma_{e-1}} r^2 dr = (1 - \alpha) E_0,$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 52-55, January-February, 1978. Original article submitted December 27, 1976.



taking into account the redistribution of the energy evolved between heat and motion. Here  $\alpha$  is the initial fraction of the kinetic energy defined by the properties of the material released;  $r_{T_{0}}$  is the initial radius of the thermal wave.

In an isothermal shock compression,  $r = r_1$ , the relations

$$\begin{cases} [\rho (D-v)] = 0, \\ [\rho v (D-v) - \rho a^{2}] = 0, \\ \left[ \rho (D-v) \left( \frac{v^{2}}{2} + \frac{a^{2}}{\gamma_{e} - 1} \right) - S - \rho a^{2} v \right] = 0. \end{cases}$$

are valid, where the brackets denote the differences in the corresponding values at the shock front.

Analysis of the dimensions of the defining parameters shows that the problem is not selfsimilar. We shall solve it by a method similar to that in [1].

We shall assume that the temperature is constant over the whole heated region

$$T(r, t) = \begin{cases} T(t), & r \in [0, r_T], \\ 0, & r > r_T, \end{cases}$$

where  $r_T$  is the radius of the thermal wave front.

We specify the density distribution and the velocity of the gas behind the shock front

$$\rho = \rho_1 (r/r_1)^m,$$

$$v = D(1 - \rho_0/\rho_1)(r/r_1), r \in [0, r_1].$$
(6)

Multiplying Eq. (1) by  $4\pi r^2$  and integrating over the region of the moving gas and taking Eq. (6) into account, we obtain the expression for the coefficient m

$$m = \frac{4\pi r_1^3 \rho_1}{M_0 + 4\pi \rho_0 r_1^3/3} - 3.$$
 (7)

Then, similar to [1], we derive the system of ordinary differential equations for the approximate calculation of the propagation of the internal shock and thermal waves

$$dr_1/dt = D, (8)$$

where D satisfies the condition

$$D = \left[\frac{2E_{\rm k}}{\left(M_0 + 4\pi\rho_0 r_1^3/3\right)(1 - a^2/D^2)} + \frac{E_{\rm k}}{\pi r_1^3\rho_1\left(1 - a^2/D^2\right)}\right]^{1/2},$$
  
$$\rho_1 = \rho_0 \frac{D^2}{a^2}; \tag{9}$$

$$\frac{dr_T}{dt} = \frac{S_T(\gamma_e - 1)}{\rho_0 A_0 T},$$
(10)

where  $S_{T} = (7.52/r_{T})(T/10^{6})^{2} \sigma T^{4}$  is the radiation flow at the thermal wave front;

$$T = \frac{(\gamma_{e} - 1)(E_{0} - E_{k})}{A_{e}(M_{0} + 4\pi\rho_{0}r_{T}^{3}/3)};$$
(11)

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$$\frac{dE_{\mathbf{k}}}{dt} = 4\pi r_1^2 \rho_0 D^3 \bigg[ \frac{3\left(1 - a^2/D^2\right)}{3 + m} - \frac{\left(1 - a^4/D^4\right)}{2} \bigg],\tag{12}$$

where  $E_k = 4\pi \int_0^{\infty} \frac{\rho v^2}{2} r^2 dr$  is the kinetic energy of the moving gas.

We note that when  $M_0 \rightarrow 0$ , systems (7)-(12) convert to the corresponding equations in [1]. From the conditions (6) when m = 0 and  $\rho_0/\rho_1 \ll 1$ , it follows that the initial dispersion of the mass released takes place under the self-similar conditions considered in [3]. When m = 0, the distributions (6) are satisfied by the equation of continuity (1) for any function  $r_1(t)$ .

As initial values for the problem of numerical computation when t = 0, we take  $r_{T_0}$  = 30 m,  $r_1 = 0$ ,  $\alpha = 0.5$ , m = 0,  $\rho_0 = 1.29 \text{ kg/m}^3$ . Consequently, for t = 0,  $\rho_1 r_1^3 = 3M_0/4\pi$ .

Figure 1 shows the relations between the radii of the thermal and shock waves and the time. Curves 1-3 correspond to the values  $M_0 = 0$  (self-similar solution [4]), 0.1, and 10 t. It can be seen that the mass released depends significantly on the conditions of propagation of the thermal and shock waves.

## LITERATURE CITED

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DEVELOPMENT OF DYNAMIC PERTURBATIONS IN INITIAL STAGE OF A POINT EXPLOSION IN A THERMALLY CONDUCTING GAS

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UDC 533.6.011,534.222.2

The one-dimensional perturbations originating in a cold homogeneous gas  $(T_1 = 0, \rho_1 = \text{const})$  by the instantaneous release of finite energy at the origin of the coordinates are considered. The starting equations are compiled for a gas in which the heat-transfer mechanism is simulated by a nonlinear thermal conductivity with coefficient  $\lambda \sim T^n$ . Transformation of the equations to the dimensionless form by the introduction of "natural" variable allows the simplest path for investigating the process as a whole to be shown by means of the method of perturbations. The initial approximation corresponds to the well-known solution for a thermal wave [1], while subsequent approximations describe the joint development of both thermal and dynamic perturbations. An investigation of the properties of the solutions and an example of the calculation of the first two approximations (without taking account of the starting approximation) for the case of a point spherical explosion with n = 5 gives a representation of the formation of the shock wave.

When studying an explosion in a gas, it is of great importance to take into account the actual heat-transfer processes. This is especially important in the very first stage of the explosion or, as observations and theoretical investigations [2] show, the thermal wave originates during the explosion even before the appearance of the dynamic nature of the phenomenon. The heat-transfer mechanism, in this case, is due mainly to the effect of radiation, but if we neglect the pressure and the radiation energy, then a completely acceptable

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 55-62, January-February, 1978. Original article submitted December 24, 1976.